# Kennesaw State University

College of Computing and Software Engineering

Department of Computer Science

CS 5070, Mathematical Structures for Computer Science, Section W01

Assignment #4

Amrit Singh, [asingh59@students.kennesaw.edu](mailto:asingh59@students.kennesaw.edu)

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## Problem Statement

Problem of this document is to solve the following problem set for assignment #4.

Module 4 Assignment #4 (Sequences)

1. Problem 2.1.10 (page 145)
2. Problem 2.2.3 (page 156)
3. Problem 2.3.8 (page 165)
4. Problem 2.4.7 (page 176)

5. Problem 2.5.7 (page 188)

## Summary / Purpose

Purpose of this document is to provide solutions to the problem set outlined in the Problem Statement section.

## Solutions

1. Problem 2.1.10 (page 145) – Show that is also a solution to the recurrence relation . What would the initial conditions need to be for this to be the closed formula for the sequence?

A closed formula for a sequence is a formula for using a fixed finite number of operations on n. A recursive relation is an equation relating a term of the sequence to previous terms (terms with smaller index) and an initial condition.

For

If n = 1;

If n = 2;

If n = 3;

Then,

Therefore, is a solution to the reoccurrence relation .

For it to be a closed formula, we need to find To do so, we can find substitute in for in the reoccurrence relation.

Therefore our initial condition is when .

1. Problem 2.2.3 (page 156) – Consider the sum 4 + 11 + 18 + 25 + … + 249
   1. How many terms (summands) are in the sum?

The summation here is an arithmetic sequence because each summand differs by a constant, with that constant being 7, with initial term being 4. Therefore, we can find the number of terms by substituting in:

But note that n here is at the a1 term, 4 is a0 term so in total there are 36 terms. Therefore, there are 36 terms in the summation.

* 1. Compute the sum using a technique discussed in this section.

Add the first and last terms of the sequence, and then reverse. Let’s call the sum *S*. See:

*S* = 4 + 11 + 18 + 25 + … + 242 + 249

*S* = 249 + 242 + 235 + 228 + … + 11 + 4

2*S* = 253 + 253 + 253 + 253 + … + 253 + 253

We know there are 35 terms in the summation. So:

2*S* = 36\*253 = 9108

*S* = 4554

Therefore, the sequence is equal to 4554.

1. Problem 2.3.8 (page 165) – Suppose . Find a closed formula for the sequence of differences by computing .

For

If n = 0; = 4

If n = 1;

If n = 2;

If n = 3; = 22

If n = 4; = 32

We have the sequence of 4, 8, 14, 22, 32. The sequence of differences is: 4, 6, 8, 10. Therefore, a closed formula for the sequence of differences is 2n + 2.

We can also substitute:

So:

1. Problem 2.4.7 (page 176) – Solve the reoccurrence relation with initial terms and

A reoccurrence relation is a recursive definition without the initial conditions. The characteristic equation is of the form . Let’s factor this to get the quadratic equation:

The roots are then and

The general solution is of the following form:

Substitute the roots in to get:

Now let’s use the initial conditions to find c1 and c2.

Solving,

Plugging into the first equation,

So,

Substituting into the general equation, we get our solution to the reoccurrence relation:

1. Problem 2.5.7 (page 188) – Prove by mathematical induction, that , where is the *n*th Fibonacci number ( and ).

The Fibonacci sequence is a series of numbers where each number is the sum of the two preceding ones, typically starting with 0 and 1. So, the sequence begins 0, 1, 1, 2, 3, 5, 8, 13, and so on.

The base case (n = 0) is: F0 = F2 – 1 => F0 = 1 – 1 = 0, which is true for n = 0.

Now using induction,

Let’s determine this by solving for :

Solving the right-hand side:

=>

=>

Therefore,

Using this, we can determine that the sequence

Is true for all non-negative integers n.

## References

[1] Garrido, J. (2021, August 14). *CS5070 Mathematical Structures for Computer Science - Notes 4* [Slide show; Powerpoint]. D2L. https://kennesaw.view.usg.edu/d2l/le/content/3550928/viewContent/55786717/View?ou=3550928

[2] Kennesaw State University, College of Computing and Software Engineering, Department of Computer Science, Mathematical Structures for Computer Science. (n.d.). CS5070 Assignment 4. In *https://kennesaw.view.usg.edu/d2l/le/content/3550928/viewContent/55786683/View*

[3] Levin, O. (2016). *Discrete mathematics: An Open Introduction*.